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Tunneling into a quantum Hall system and the infrared catastrophe

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Abstract

We calculate the tunneling density of states for a two-dimensional interacting electron gas in a quantizing magnetic field. We show that the observed pseudogap in the density of states of the lowest Landau level can be regarded to be a consequence of an infrared catastrophe, resulting from the response of the electron system to the potential produced by the abruptly added charge during a tunneling event. Our formalism can be applied at any filling factor without the use of Chern–Simons or composite fermion theory.

1. Introduction

Systems where interactions dominate are intrinsically interesting, but are also difficult to treat theoretically. A classic example of this is the quantum Hall system, where the dynamics are almost if not entirely driven by interactions and where specialized nonperturbative theoretical techniques have to be used. These interactions lead to the nontrivial conductance quantization of the quantum Hall effect. One would expect that the tunneling properties (i.e., tunneling density of states) of a quantum Hall system would be influenced by the strong interactions as well. In fact, experiments [1] show that the low-energy tunneling density of states (DOS) develops a pseudogap when the lowest Landau level (LLL) is partially filled. Several theoretical techniques including Chern–Simons theory as well as standard albeit sophisticated diagrammatic approaches have reproduced the pseudogap behavior [2–7]. These specialized approaches are necessary because the strong interactions between particles make perturbative methods inapplicable. Here we take a different approach where the physics of the tunneling event itself plays the primary role.

Many systems exhibit a suppression in the DOS at the Fermi energy. In previous papers we proposed that the underlying physics of this suppression is the infrared catastrophe (IRC) caused by the sudden introduction of a new localized electron into the host system during tunneling [8–10]. In systems where the accommodation of the new electron is inhibited by dimensionality, applied perpendicular magnetic fields, or disorder, one would expect an IRC to occur,

analogous to that of the x-ray edge problem [11]. In a quantum Hall experiment it is the strong magnetic field that suppresses the recoil of the tunneling electron. In the limit that the recoil of the new electron is fully suppressed, the potential it produces is of the form (assuming the electron is added (tunnels) at time $t = 0$ and removed at t_0)

$$\phi_{\text{xt}}(\mathbf{r}, t) = U(\mathbf{r})\Theta(t_0 - t)\Theta(t) \quad (1)$$

where $U(\mathbf{r})$ is the electron–electron interaction and Θ is the unit step function.

In this work we show that the response of the host system to a potential of the form (1) can explain the experimentally observed pseudogap. Previously, we applied this approach to the LLL using the simplest of interactions, a delta function [8, 9]. In [8] and [9] we obtained an energy gap in the DOS instead of a pseudogap. In this paper we use the actual (screened) Coulomb interaction for $U(\mathbf{r})$ and recover the observed pseudogap. Our result suggests that the pseudogap observed in [1] can be understood as another example of the more general infrared catastrophe phenomena, which also determines the DOS in one-dimensional electron systems [8, 9]. In addition, our approach allows one to determine the relationship between the form of the electron–electron interaction $U(\mathbf{r})$ and the tunneling DOS in a straightforward manner, an example of which is given below.

Our work concerns the DOS and associated tunneling spectrum of a single-layer two-dimensional electron gas in the quantum Hall regime, both at compressible and incompressible

filling factors. Experiments on both single-layer and double-layer systems are possible, the main difference being the presence of a so-called excitonic shift of the peak of the DOS downward, caused by the attraction between a tunneling electron and the hole it leaves behind [1]. Most of the early theoretical work on this problem also assumed a single layer [2–5, 7], but Klironomos and Dorsey [6] have generalized the Wigner crystal model of Johansson and Kinaret [4] to a double-layer system.

2. General formalism

The general formalism is applicable to a wide variety of systems and has been discussed in detail in our prior work [8–10]. For completeness we restate the general outline here. Note that in our previous work we used the Euclidean time formalism, but here—to avoid a later difficulty in analytic continuation—we work in real time.

Starting with a general D -dimensional interacting electron system, including a possible external magnetic field, the Hamiltonian is written as

$$H = \sum_{\sigma} \int d^D r \Psi_{\sigma}^{\dagger}(\mathbf{r}) \left[\frac{\Pi^2}{2m} + v_0(\mathbf{r}) - \mu_0 \right] \Psi_{\sigma}(\mathbf{r}) + \frac{1}{2} \sum_{\sigma\sigma'} \int d^D r d^D r' \Psi_{\sigma}^{\dagger}(\mathbf{r}) \Psi_{\sigma'}^{\dagger}(\mathbf{r}') U(\mathbf{r} - \mathbf{r}') \times \Psi_{\sigma'}(\mathbf{r}') \Psi_{\sigma}(\mathbf{r}), \quad (2)$$

where $\Pi \equiv \mathbf{p} + \frac{e}{c} \mathbf{A}$, and where $v_0(\mathbf{r})$ is any single-particle potential energy, which may include a periodic lattice potential or disorder or both. Apart from an additive constant we can write H as $H_0 + V$, where

$$H_0 = \sum_{\sigma} \int d^D r \Psi_{\sigma}^{\dagger}(\mathbf{r}) \left[\frac{\Pi^2}{2m} + v(\mathbf{r}) - \mu \right] \Psi_{\sigma}(\mathbf{r}) \quad (3)$$

and

$$V = \frac{1}{2} \int d^D r d^D r' \delta n(\mathbf{r}) U(\mathbf{r} - \mathbf{r}') \delta n(\mathbf{r}'). \quad (4)$$

H_0 is the Hamiltonian in the Hartree approximation. The single-particle potential $v(\mathbf{r})$ includes the Hartree interaction with self-consistent density $n_0(\mathbf{r})$,

$$v(\mathbf{r}) = v_0(\mathbf{r}) + \int d^D r' U(\mathbf{r} - \mathbf{r}') n_0(\mathbf{r}'), \quad (5)$$

where

$$n_0(\mathbf{r}) = \left\langle \sum_{\sigma} \Psi_{\sigma}^{\dagger}(\mathbf{r}) \Psi_{\sigma}(\mathbf{r}) \right\rangle_0, \quad (6)$$

and the chemical potential in H_0 has been shifted by $-U(0)/2$. Here $\langle O \rangle_0 = \text{Tr}(e^{-\beta H_0} O) / \text{Tr}(e^{-\beta H_0})$ denotes an expectation value with respect to the Hartree-level Hamiltonian. In a translationally invariant system the equilibrium density is unaffected by interactions, but in a disordered or inhomogeneous system it will be necessary to distinguish between the approximate Hartree and the exact equilibrium density distributions. The interaction in (3) is written in terms of the density fluctuation

$$\delta n(\mathbf{r}) \equiv \left[\sum_{\sigma} \Psi_{\sigma}^{\dagger}(\mathbf{r}) \Psi_{\sigma}(\mathbf{r}) \right] - n_0(\mathbf{r}) = n(\mathbf{r}) - n_0(\mathbf{r}). \quad (7)$$

We want to calculate the zero-temperature time-ordered propagator

$$G(\mathbf{r}_f \sigma_f, \mathbf{r}_i \sigma_i, t_0) \equiv -i \langle T \Psi_H(\mathbf{r}_f \sigma_f, t_0) \Psi_H^{\dagger}(\mathbf{r}_i \sigma_i, 0) \rangle_H \quad (8)$$

for the interacting system, which can be written (in the interaction representation with respect to H_0) as

$$G(\mathbf{r}_f \sigma_f, \mathbf{r}_i \sigma_i, t_0) = -i \frac{\langle T \Psi(\mathbf{r}_f \sigma_f, t_0) \Psi^{\dagger}(\mathbf{r}_i \sigma_i, 0) e^{-i \int_{-\infty}^0 dt V(t)} \rangle_0}{\langle T e^{-i \int_{-\infty}^0 dt V(t)} \rangle_0}. \quad (9)$$

Performing a Hubbard–Stratonovich transformation of the form

$$e^{-\frac{1}{2} \int \delta n U \delta n} = \frac{\int D\phi e^{\frac{1}{2} \int \phi U^{-1} \phi} e^{-i \int \phi \delta n}}{\int D\phi e^{\frac{1}{2} \int \phi U^{-1} \phi}} \quad (10)$$

leads to

$$G(\mathbf{r}_f \sigma_f, \mathbf{r}_i \sigma_i, t_0) = \mathcal{N} \frac{\int D\phi e^{\frac{1}{2} \int \phi U^{-1} \phi} g(\mathbf{r}_f \sigma_f, \mathbf{r}_i \sigma_i, t_0 | \phi)}{\int D\phi e^{\frac{1}{2} \int \phi U^{-1} \phi}}, \quad (11)$$

where

$$g(\mathbf{r}_f \sigma_f, \mathbf{r}_i \sigma_i, t_0 | \phi) = -i \langle T \Psi(\mathbf{r}_f \sigma_f, t_0) \Psi^{\dagger}(\mathbf{r}_i \sigma_i, 0) e^{-i \int dt \int d^D r \phi(\mathbf{r}, t) \delta n(\mathbf{r}, t)} \rangle_0 \quad (12)$$

is a noninteracting correlation function, and $\mathcal{N} \equiv \langle T e^{-i \int_{-\infty}^0 dt V(t)} \rangle_0^{-1}$ is a constant (independent of t_0). So far no approximations have been made. To make any progress one has to determine what the important field configurations in (11) are and how to treat them.

In systems where the recoil of the newly added electron is suppressed by applied fields, disorder, dimensionality, or any combination thereof, we propose that the important field configurations are those close to ϕ_{xr} . These fields correspond to potentials of recoilless electrons being added to the system.

If we neglect all fields except ϕ_{xr} in (11), the so-called x-ray edge limit, the fully interacting Green's function (8) is given by

$$G(\mathbf{r}_f \sigma_f, \mathbf{r}_i \sigma_i, t_0) \approx \mathcal{N} g(\mathbf{r}_f \sigma_f, \mathbf{r}_i \sigma_i, t_0 | \phi_{\text{xr}}). \quad (13)$$

Next we define the Green's function

$$G_{\text{xr}}(\mathbf{r}\sigma t, \mathbf{r}'\sigma' t') \equiv -i \frac{\langle T \Psi(\mathbf{r}'\sigma', t') \Psi^{\dagger}(\mathbf{r}\sigma, t) e^{-i \int dt \int d^D r \phi_{\text{xr}}(\mathbf{r}, t) n(\mathbf{r}, t)} \rangle_0}{Z_{\text{xr}}} \quad (14)$$

where $Z_{\text{xr}} = \langle T e^{-i \int dt \int d^D r \phi_{\text{xr}}(\mathbf{r}, t) n(\mathbf{r}, t)} \rangle_0$. The correlation function in (13) can be written in terms of G_{xr} and Z_{xr} as

$$g(\mathbf{r}_f \sigma_f, \mathbf{r}_i \sigma_i, t_0 | \phi_{\text{xr}}) = Z_{\text{xr}} G_{\text{xr}}(\mathbf{r}_f \sigma_f t_0, \mathbf{r}_i \sigma_i 0) e^{i \int dt \int d^D r \phi_{\text{xr}}(\mathbf{r}, t) n_0(\mathbf{r})}. \quad (15)$$

Thus the full Green's function in the x-ray edge limit is

$$G(\mathbf{r}_f \sigma_f, \mathbf{r}_i \sigma_i, t_0) = \mathcal{N} Z_{\text{xr}} G_{\text{xr}}(\mathbf{r}_f \sigma_f t_0, \mathbf{r}_i \sigma_i 0) e^{i \int dt \int d^D r \phi_{\text{xr}}(\mathbf{r}, t) n_0(\mathbf{r})}. \quad (16)$$

As discussed above, the field $\phi_{\text{xr}}(\mathbf{r}, t)$ defined in (1) describes the potential that would be produced by an electron added to the system at $\mathbf{r} = 0$ and then subsequently removed.

While sitting at the origin the ‘electron’ does not move, as though it had an infinite mass. Thus the x-ray edge limit neglects recoil of the tunneling electron, an approximation which is valid here because of the strong magnetic field, which confines any added charge to a region of a size of the order of a magnetic length. We note that although our approach is not able to correctly account for the effects of strong correlation on the two-particle properties (such as the compressibility) of the quantum Hall system, the *one*-particle spectral properties are in fact accurately described, as is the case for the 1D interacting electron gas [8, 9].

3. Application to the LLL

Here we apply the above the formalism to the spin-polarized LLL. A Dyson equation is solved in section 3.1 for G_{xr} . Z_{xr} is then calculated in section 3.2. These two factors give the DOS in the x-ray edge limit, (16).

3.1. Dyson equation

G_{xr} satisfies a Dyson equation given by

$$G_{\text{xr}}(\mathbf{r}, \mathbf{r}', t, t') = G_0(\mathbf{r}, \mathbf{r}', t, t') + \int dt'' d^2r'' G_0(\mathbf{r}, \mathbf{r}'', t - t'') \phi_{\text{xr}}(\mathbf{r}'', t'') \times G_{\text{xr}}(\mathbf{r}'', \mathbf{r}', t'', t'). \quad (17)$$

Choosing $\mathbf{B} = -B\mathbf{e}_z$ and the symmetric gauge $\mathbf{A} = By/2\mathbf{e}_x - Bx/2\mathbf{e}_y$, the noninteracting Green’s function projected into the LLL is

$$G_0(\mathbf{r}, \mathbf{r}', t) = i \sum_{m=0}^{\infty} \phi_m(\mathbf{r}) \phi_m^*(\mathbf{r}') [v - \Theta(t)] \quad (18)$$

where the

$$\phi_m(\mathbf{r}) = \frac{r^m}{\sqrt{2\pi} 2^m m!} e^{-r^2/4} e^{im\theta} \quad (19)$$

are the noninteracting single-particle eigenfunctions of the LLL and $0 < v < 1$ is the filling factor. We work in units where $\hbar = \ell = 1$. Here $\ell = \sqrt{\hbar c/eB}$ is the magnetic length. Furthermore, we assume a screened Yukawa potential of the form

$$U(\mathbf{r}) = \frac{e^2}{r} e^{-r/\alpha}, \quad (20)$$

with α a screening length determined by the distance to the nearest gate. The potential (20) is diagonal in the basis (19) with matrix elements

$$\int d^2r \phi_m^*(\mathbf{r}) U(\mathbf{r}) \phi_{m'}(\mathbf{r}) = \lambda_m \delta_{m,m'}, \quad (21)$$

where

$$\lambda_m(\alpha) \equiv \frac{e^2}{m!} \left[\frac{1}{\sqrt{2}} \Gamma(m + \frac{1}{2}) \Phi(m + \frac{1}{2}, \frac{1}{2}; \frac{1}{2\alpha^2}) - \frac{m}{\alpha} \Gamma(m) \Phi(m + 1, \frac{3}{2}; \frac{1}{2\alpha^2}) \right]. \quad (22)$$

Here Γ is the gamma function and Φ is the confluent hypergeometric function. To solve (17) we use the ansatz

$$G_{\text{xr}}(\mathbf{r}, \mathbf{r}', t, t') = \sum_m a_m(t, t') \phi_m(\mathbf{r}) \phi_m^*(\mathbf{r}'). \quad (23)$$

Substituting (23) into (17) gives

$$\sum_m a_m(t, t') \phi_m(\mathbf{r}) \phi_m^*(\mathbf{r}') = i \sum_m \phi_m(\mathbf{r}) \phi_m^*(\mathbf{r}') [v - \Theta(t - t')] + ie^2 \sum_{m,l} \phi_m(\mathbf{r}) \phi_l^*(\mathbf{r}') \int_0^{t_0} dt'' [v - \Theta(t - t'')] a_m(t'', t') \times \int d^2r'' \phi_m^*(\mathbf{r}'') U(\mathbf{r}'') \phi_l(\mathbf{r}''). \quad (24)$$

Thus we need to solve the integral equation

$$a_m(t, t') = i[v - \Theta(t - t')] + i\lambda_m \int_0^{t_0} dt'' [v - \Theta(t - t'')] \times a_m(t'', t'). \quad (25)$$

Equation (25) is similar to an integral equation we have solved previously [9]. The solution is

$$a_m(t, t') = i \frac{(v - 1)\Theta(t - t') + v\Theta(t' - t)e^{-i\lambda_m t_0}}{1 - v + v e^{-i\lambda_m t_0}} e^{-i\lambda_m(t - t')}. \quad (26)$$

Using the solution of (17) we can calculate Z_{xr} .

3.2. Evaluation of Z_{xr}

From the appendix below,

$$Z_{\text{xr}} = e^M \quad (27)$$

where

$$M = - \int_0^{t_0} dt \int_0^1 d\xi \int d^D r U(\mathbf{r}) G_{\text{xr}}^\xi(\mathbf{r}, \mathbf{r}, t, t^+) \quad (28)$$

and

$$G_{\text{xr}}^\xi(\mathbf{r}, \mathbf{r}, t, t^+) = i \sum_m \frac{v e^{-i\xi \lambda_m t_0}}{1 - v + v e^{-i\xi \lambda_m t_0}} |\phi_m(\mathbf{r})|^2. \quad (29)$$

Performing the time and space integrals gives

$$M = -it_0 \sum_{m=0}^{\infty} \lambda_m \int_0^1 d\xi \frac{v e^{-i\xi \lambda_m t_0}}{1 - v + v e^{-i\xi \lambda_m t_0}}, \quad (30)$$

and finally

$$M = \sum_{m=0}^{\infty} \ln [1 - v + v e^{-i\lambda_m t_0}]. \quad (31)$$

We note that this summation diverges in the $\alpha \rightarrow \infty$ (unscreened) limit. Thus,

$$Z_{\text{xr}} = \exp \left(\sum_{m=0}^{\infty} \ln [1 - v + v e^{-i\lambda_m t_0}] \right) = \prod_{m=0}^{\infty} [1 - v + v e^{-i\lambda_m t_0}]. \quad (32)$$

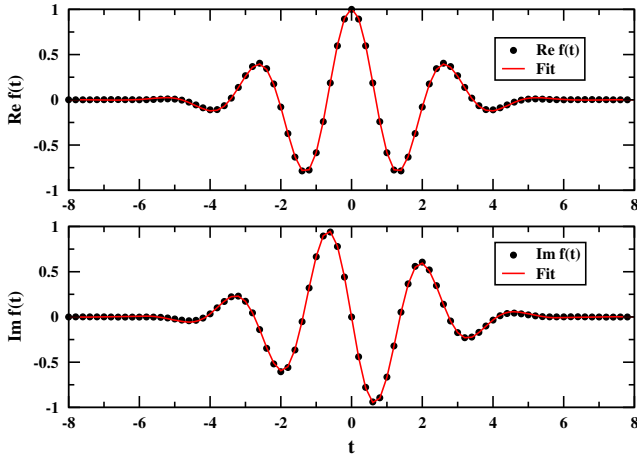


Figure 1. Example fit to the second form of equation (37) with $\alpha = 10 \ell$ and $\nu = 0.48$ [1].

(This figure is in colour only in the electronic version)

4. Results

The DOS is given by

$$N(\omega) = -\frac{1}{\pi} \text{sign}(\omega) \text{Im } G(\omega) \quad (33)$$

where $G(\omega)$ is the Fourier transform of $G(t)$

$$G(\omega) = \int_{-\infty}^{\infty} dt G(t) e^{i\omega t}. \quad (34)$$

In the x-ray edge limit $G(t)$ is

$$G(t_0) = \mathcal{N} Z_{\text{xr}}(t_0) G_{\text{xr}}(t_0) e^{i \int dt d^D r \phi_{\text{xr}}(\mathbf{r}, t) n_0(\mathbf{r})} \quad (35)$$

or

$$G(t_0) = \left[-\mathcal{N} \frac{i}{2\pi} (1-\nu) \Theta(t_0) e^{-i\lambda_0 t_0} e^{i\nu e^2 \alpha t_0} + \mathcal{N} \frac{i}{2\pi} \nu \Theta(-t_0) e^{-i2\lambda_0 t_0} e^{i\nu e^2 \alpha t_0} \right] \times \prod_{m=1}^{\infty} [1 - \nu + \nu e^{-i\lambda_m t_0}]. \quad (36)$$

The product in (36) is difficult to handle analytically so we will use a numerical fit. The product is well approximated by a modulated Gaussian with three fit parameters $\{c_1, c_2, c_3\}$,

$$f(t) = \prod_{m=1}^{\infty} [1 - \nu + \nu e^{-i\lambda_m t_0}] \approx c_1 e^{ic_2 t} e^{-c_3 t^2}. \quad (37)$$

An example of the fit is shown in figure 1.

Using the function of (37) the Fourier transform of (36) can be done analytically, such that

$$G(\omega > 0) = -i\mathcal{N} \frac{c_1}{2\pi} (1-\nu) e^{-\frac{(\omega-\omega_0)^2}{c_3(e^2/\ell)^2}}, \quad (38)$$

which is just a Gaussian centered at ω_0 with energy width $\sqrt{c_3} e^2/\ell$. This leads to a DOS (for $\omega > 0$) given by³

$$N(\omega) = \mathcal{N} \frac{c_1}{2\pi^2} (1-\nu) e^{-\frac{(\omega-\omega_0)^2}{c_3(e^2/\ell)^2}}. \quad (39)$$

³ The negative time contribution from (36) simply gives another Gaussian line-shape for $\omega < 0$.

Table 1. Results of best fit to function (37). α is the screening length and ℓ is the magnetic length.

α/ℓ	c_1	c_2	c_3
5	1.0	1.3	0.08
10	1.0	2.0	0.13
15	1.0	2.7	0.16
20	1.0	2.9	0.19

The width of the Gaussian is of the order seen experimentally [1], but the energy shift ω_0 —or more specifically the fit parameter c_2 —cannot be accurately determined. This is because short-time physics enters Z_{xr} through $G_{\text{xr}}(t, t^+)$ and we have solved (17) using a long-time approximation by including only the LLL in G_0 . This is a well known problem dating from the original solution of the x-ray edge problem [11], where the threshold energy cannot be obtained accurately but the exact x-ray edge exponent is found.

The fit parameters for two values of screening length α (see (20)) are summarized in table 1. In a single-layer system, α is given by the distance to the nearest metallic gate, whereas in a double-layer system it can be assumed to be the distance between the layers. We note that the case $\alpha = 5 \ell$ corresponds approximately to the experiment of [1], and that in the $\alpha \rightarrow 0$ limit the peak width vanishes, in agreement with [10].

5. Summary

In this paper we calculate the DOS for tunneling into the LLL using the (screened) Coulomb interaction. The calculation is done within the x-ray edge limit, which amounts to the neglect of recoil of the tunneling electron. Even in this approximation we are able to reproduce the experimentally observed pseudogap. Although the position of the conductance peak cannot be accurately determined from the present calculation, the inclusion of higher Landau levels would remedy this.

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Appendix. Construction of Z_{xr}

Using the linked cluster theorem,

$$Z_{\text{xr}} = \langle T e^{-i \int dt d^D r \phi_{\text{xr}}(\mathbf{r}, t) \hat{n}(\mathbf{r}, t)} \rangle_0 = e^M, \quad (A.1)$$

where

$$M = \sum_{l=1}^{\infty} \frac{(-i)^l}{l} \int dt_1 d^D r_1 \cdots \int dt_l d^D r_l \times \phi_{\text{xr}}(\mathbf{r}_1, t_1) \cdots \times \phi_{\text{xr}}(\mathbf{r}_l, t_l) \langle T \hat{n}(\mathbf{r}_1, t_1) \cdots \hat{n}(\mathbf{r}_l, t_l) \rangle_{\text{different connected}} \quad (A.2)$$

or

$$M = - \sum_{l=1}^{\infty} \frac{1}{l} \int_0^{t_0} dt_1 \int d^D r_1 \cdots \int_0^{t_0} dt_l \int d^D r_l \times U(\mathbf{r}_1) \cdots U(\mathbf{r}_l) G_0(\mathbf{r}_1, \mathbf{r}_l, t_1, t_l) \cdots \times G_0(\mathbf{r}_l, \mathbf{r}_1, t_l, t_1^+). \quad (\text{A.3})$$

Changing the summation limit leads to

$$M = - \sum_{l=0}^{\infty} \frac{1}{l+1} \int_0^{t_0} dt_1 \int d^D r_1 \cdots \int_0^{t_0} dt_{l+1} \int d^D r_{l+1} \times U(\mathbf{r}_1) \cdots U(\mathbf{r}_{l+1}) G_0(\mathbf{r}_1, \mathbf{r}_{l+1}, t_1, t_{l+1}) \cdots \times G_0(\mathbf{r}_{l+1}, \mathbf{r}_1, t_{l+1}, t_1^+). \quad (\text{A.4})$$

Using

$$G_{\text{xr}}^{\xi}(\mathbf{r}, \mathbf{r}', t, t^+) = G_0(\mathbf{r}, \mathbf{r}', t, t^+) + \xi \int_0^{t_0} dt'' \times \int d^D r'' G_0(\mathbf{r}, \mathbf{r}'', t, t'') U(\mathbf{r}'') G_{\text{xr}}^{\xi}(\mathbf{r}'', \mathbf{r}', t'', t^+) \quad (\text{A.5})$$

and

$$\frac{1}{l+1} = \int_0^1 d\xi \xi^l. \quad (\text{A.6})$$

Equation (A.4) can be written simply as

$$M = - \int_0^{t_0} dt \int_0^1 d\xi \int d^D r U(\mathbf{r}) G_{\text{xr}}^{\xi}(\mathbf{r}, \mathbf{r}, t, t^+). \quad (\text{A.7})$$

Reinstating spin for completeness gives

$$M = - \sum_{\sigma} \int_0^{t_0} dt \int_0^1 d\xi \int d^D r U(\mathbf{r}) G_{\text{xr}}^{\xi}(\mathbf{r}, \mathbf{r}, t, t^+, \sigma). \quad (\text{A.8})$$

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